national accelerator laboratory

NAL-PUB-73/44-THY
June, 1973

TESTS OF HIGHER SYMMETRIES

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ABSTRACT

Model-independent cross section relations predicted by unbroken SU(3) symmetry, and some predicted by $SU(6)_{W,strong}$, are compared with experiment. The relations are found to be satisfied, apart from deviations which follow, in every case, the pattern and rough size of symmetry breaking expected from Regge pole exchange. Interestingly, this Regge symmetry breaking diverges with increasing energy. It is argued that this behavior, though contrary to intuition, is reasonable.

*Work supported in part by the National Science Foundation

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I. INTRODUCTION

In the limit of unbroken SU(3) symmetry, there are certain two-body (and quasi two-body) reactions whose amplitudes are predicted to be equal, purely as a consequence of the symmetry, and independent of any dynamical details.[1,2] Comparisons between the cross sections for such processes serve as particularly clean tests of SU(3), and also as tests of various hypotheses for symmetry breaking. In this paper we report on several such comparisons, and on the test of a triangle inequality, involving meson-baryon reactions. The cross sections we consider are found to obey the SU(3) relations, apart from factors of 2 or 3 which, in every case, are qualitatively consistent with the symmetry breaking expected from Regge pole exchange. We also report on two analogous tests of $SU(6)_{W,\mathrm{strong}}$, with similar results.

Apart from those involved in the triangle inequality, all the reactions we deal with are exotic in the t channel. As a result, their cross sections are relatively small at energies high enough to get away from threshold and mass difference effects. However, these reactions are not exotic in the u channel. Thus, in the backward direction they should be governed by exchange of ordinary baryons, and have cross sections comparable to those of other backward meson-baryon processes. Now that better data are becoming available for backward scattering, more meaningful, though still crude, tests of the long- known SU(3) equalities[1,2] become possible.

Testing of the SU(3) equalities in the backward direction also has the advantage that since one has some idea of the underlying dynamics in this region, one can estimate the manner in which the exact symmetry predictions, which apply to any angle and are independent of dynamics, will be broken. In the forward direction one knows very little about the dynamics, because of the exotic t channel, so it would be very difficult to guess how the symmetry is broken.

Suppose we compare reactions whose cross sections are supposed to be equal or proportional, and which are governed in the backward direction by exchange of baryon Regge poles belonging to some SU(3) multiplet. Then the cross section for a reaction involving exchange of the non-strange member of the multiplet will be proportional to $(s/s_o)^{2\alpha_N(u)-2}$, where α_N is the non-strange trajectory, and s_o is a scale factor. By comparison, the cross section for a reaction involving strange exchange will be proportional to $(s/s_o)^{2\alpha_S(u)-2}$, where α_S is the strange trajectory. Thus, at a common s and u, the two cross sections will be approximately in the ratio

$$(s/s_o)^{2(\alpha_N(u) - \alpha_S(u))}, \tag{1.1}$$

relative to the ratio expected from unbroken SU(3). In saying this, we are assuming that, as indicated by previously obtained evidence,[3] the Regge <u>residues</u> are SU(3) invariant. We are also neglecting some kinematical effects.

It is understood, of course, that the specific factor (1.1) is only a qualitative guide to the symmetry breaking to be expected. This factor may fail quantitatively for numerous reasons: (1) the value of s_o or of $\alpha_N - \alpha_S$ might be very different from what one guesses; (2) there might be several Regge multiplets contributing significantly (with different energy-dependences); (3) signature factors can contribute to the symmetry breaking (such factors are absent, however, to the extent that exchange degeneracy holds in reactions with exotic t channels, such as those we consider); (4) there could be Regge cuts which dominate over the Regge poles in some reactions. Despite these considerations, however, one still expects (1.1) to be a qualitative guide.

If one expresses the trajectories in (1.1) in terms of the masses M_N and M_S of the baryons to which they correspond, (1.1) takes the form

$$(s/s_o)^{2\alpha'(M_S^2 - M_N^2)}, \tag{1.2}$$

in which $\alpha' \approx 1\ \text{GeV}^{-2}$ is the universal slope of all trajectories. Thus, since in all the low-lying baryon multiplets the strange particles are heavier than the non-strange ones, one predicts that the reactions with strangeness exchange will always be suppressed relative to those with non-strange exchange. The exact degree of suppression depends on which Regge pole multiplets dominate in any particular case, since mass differences and scale factors can vary from multiplet to multiplet. As explained in Sec. III, (1.1) or (1.2) most accurately expresses the degree of symmetry breaking when s_o is so chosen that most of the u-dependence of the reaction amplitudes is in the factor $(s/s_o)^{\alpha(u)}$, with a minimum of u-dependence remaining in the residue $\beta(u)$. Without detailed data analysis, the s_o which accomplishes this is undetermined.

When, in contrast to what we do here, one compares reactions whose Regge exchanges are SU(3)-related, but whose overall amplitudes are not predicted to be equal by the unbroken symmetry,[3] the ratio between the cross sections depends, not only on the symmetry breaking factor (1.1), but also on factors involving not-very-well-known d/f ratios. By dealing with reactions whose amplitudes are predicted to be the same, we can now test the Regge symmetry breaking factor (1.1) in a context where the unknown d/f ratios, and hence also their possible variation with position along the trajectory, are irrelevant.

II. EXPERIMENTAL TESTS

Unbroken SU(3) predicts[1] the amplitude equalities

$$A(\pi^- p \to \Sigma^- K^+) = A(K^- n \to \Xi^- K^o) \tag{2.1}$$

$$A(K^- p \to \Sigma^- \pi^+) = A(K^- p \to \Xi^o K^o) \tag{2.2}$$

In Figure 1 are plotted the backward cross sections for the reactions of Eq. (2.1) at 2.6 GeV/c;[4,5] one sees that the two angular distributions are indeed rather similar. Although this energy, the highest at which this cross section comparison is presently possible, is a rather low one at which to invoke Reggeism, we nevertheless assume that the latter will provide a rough guide to the symmetry breaking. We expect Eq. (2.1) to be close to unbroken since both reactions involve strangeness exchange. The plotted angular distributions are consistent with this prediction. They also give some feeling for the quality of available data. In Table I, we present integrated backward cross sections for the reactions of (2.1) and (2.2) at 2.6 GeV/c;[6,7] the quoted values include only those data bins which appear to be part of the backward peripheral Regge peak. By contrast with the integrated cross sections for $\pi^- p \rightarrow \Sigma^- K^+$ and $K^- n \rightarrow \Xi^- K^o$ (which are consistent with small symmetry breaking), those for $K^- p \rightarrow \Sigma^- \pi^+$ and $K^- p \rightarrow \Xi^o K^o$ show the process with strange exchange to be suppressed relative to that with non-strange exchange, as expected from Reggeism. With α_N and α_S taken to be the nucleon and sigma trajectories, respectively, and $s_o \simeq 1$ GeV^2, (1.1) predicts that at 2.6 GeV/c, $\sigma(K^- p \rightarrow \Sigma^- \pi^+)/\sigma(K^- p \rightarrow \Xi^o K^o) \approx 4$, consistent with the observations.

The reader may notice that the process $K^- p \rightarrow \Sigma^- \pi^+$ is just the u channel of $\pi^- p \rightarrow \Sigma^- K^+$. Further, the process $K^- p \rightarrow \Xi^o K^o$ is equivalent under isospin reflection to $\overline{K^o} n \rightarrow \Xi^- K^+$, which is the u channel of $K^- n \rightarrow \Xi^- K^o$. Thus, from crossing, if the SU(3) equality between the amplitudes for $\pi^- p \rightarrow \Sigma^- K^+$ and $K^- n \rightarrow \Xi^- K^o$ were <u>exact</u>, the amplitudes for $K^- p \rightarrow \Sigma^- \pi^+$ and $K^- p \rightarrow \Xi^o K^o$ would also have to be equal, and the comparison between the latter two reactions would not be an independent test of the symmetry. In the real world, of course, the comparisons between $\pi^- p \rightarrow \Sigma^- K^+$ and $K^- n \rightarrow \Xi^- K^o$ and between $K^- p \rightarrow \Sigma^- \pi^+$ and $K^- p \rightarrow \Xi^o K^o$ do provide independent confirmations of SU(3).

Let us now turn to the quasi two-body reactions listed in Table II. In unbroken SU(3) one can show,[2] either via a simple U spin argument or by noting that all these processes are pure 27 in the t channel, that the amplitudes for these reactions are all proportional. Taking into account Clebsch-Gordan coefficients, the cross sections are related by[2]

$$\begin{aligned} 1/3\ \sigma(\pi^+ n \to \Delta^{++}\pi^-) &= \sigma(K^- p \to Y^{*-}\pi^+) = \\ \sigma(\pi^- p \to Y^{*-}K^+) &= \sigma(K^- p \to \Xi^{*-}K^+) \end{aligned} \tag{2.3}$$

The measured values of these cross sections[8] at 2 GeV/c in the backward direction are given in Table II. (In formulating the Table, we have used only the backward-most two or three data bins in each case. Also, note that for the reaction $\pi^+ n \to \Delta^{++}\pi^-$ the Table lists $1/3\ \sigma(\pi^+ n \to \Delta^{++}\pi^-)$). If SU(3) were exact, all entries in the middle column of the Table would agree. What we see, however, is that the entries for the reactions with non-strange exchange agree, and those for the reactions with strange exchange agree, but the strange exchanges are suppressed somewhat relative to the non-strange exchanges. This, of course, is just what one expects from Reggeism. With α_N and α_S taken to be the nucleon and lambda trajectories, respectively, and $s_o \simeq 1\ \mathrm{GeV}^2$, (1.1) predicts that at 2 GeV/c the strange exchanges should be suppressed by approximately a factor of 3. To see if this is correct, we multiply the strange exchange cross sections in Table II by 3, leaving the entries for non-strange exchange unchanged, and so obtain the final column in the Table. One sees that, to within errors, all the entries in this column are in agreement.[9]

It is possible, of course, for some non-strange exchange reaction to involve several large contributions which happen to cancel one another at some particular energy, leading to an anomalously small cross section at that point. In that case, the prediction of (1.1) that non-strange exchange processes have larger cross sections than those with strange exchange could fail. We observe in Tables I-II that this anomalous behavior does not occur.

In addition to the simple relations such as (2.1)-(2.3), unbroken SU(3) also yields amplitude relations involving three reactions at once, such as[1,10,11]

$$A(\pi^- p \to p\pi^-) - A(K^- p \to pK^-) = A(K^- p \to \Sigma^+\pi^-). \tag{2.4}$$

In the backward direction this relation simplifies to the approximate prediction

$$\sigma(\pi^- p \to p\pi^-) \approx \sigma(K^- p \to \Sigma^+\pi^-), \tag{2.5}$$

in consequence of the experimental fact that in that direction $\sigma(K^- p \to pK^-) \ll \sigma(\pi^- p \to p\pi^-)$.[12] Both processes in (2.5) involve non-strange baryon exchange, so one does not expect this prediction to be broken.

The relation (2.5) has already been verified at 4 GeV/c by Barger, Halzen, and Olsson,[13] who obtained it assuming the reactions to be dominated by Δ Regge pole exchange (nucleon exchange is forbidden). However, from our present point of view, we note that this relation is expected to hold even if the amplitudes are much more complicated than a single Δ Regge pole. All that is needed is the fact that the 27 u channel amplitude (which determines backward $K^- p \to pK^-$) is small, and SU(3).

In unbroken $SU(6)_{W,\text{strong}}$, there are clean predictions analogous to (2.1)-(2.3), expected to hold at 0° or 180° only. In particular, it is predicted[14] that for the processes of Table III

$$\sigma(\pi^- p \to \Sigma^- K^+) = (1/4)\ \sigma(K^- p \to \Xi^- K^+) = \sigma(K^- p \to \Sigma^- \pi^+) \tag{2.6}$$

Since these processes are all exotic in the t channel, we compare them at 180°, where the cross sections are larger and the dynamics better understood. The cross sections at 4 GeV/c[4,15,16] are given in Table III. (Note that for the reaction $K^- p \to \Xi^- K^+$ the Table lists 1/4 of the measured cross section.) If one speculates that the predictions will be broken by the same factor as breaks SU(3), i.e., (1.1), then $\sigma(\pi^- p \to \Sigma^- K^+)$ and $(1/4)\ \sigma(K^- p \to \Xi^- K^+)$ should agree with each other but should be suppressed relative to $\sigma(K^- p \to \Sigma^- \pi^+)$. Indeed, this is exactly what one sees. If we again identify α_N and α_S as the nucleon and lambda trajectories and take $s_o \simeq 1\ \text{GeV}^2$, (1.1) predicts suppression by a factor of ≈ 4.5, which is not inconsistent with experiment.

III. ENERGY DEPENDENCE OF SYMMETRY BREAKING

We have seen that the Regge factor (1.1) qualitatively describes the observed breaking of SU(3). However, as the reader may have noticed, this symmetry breaking factor has a peculiar property. Namely, it predicts that as the energy increases, ratios between SU(3)-related cross sections will go to <u>infinity</u>. Intuitively, one might expect that, on the contrary, such ratios should approach <u>unity</u> (or a Clebsch-Gordan ratio) as the energy becomes large compared to particle masses. We wish to argue that Regge symmetry breaking actually does not contradict this intuition, once one has carefully understood what is going on.

Let us look closely at the behavior expected in the Regge picture. In particular, let us explore a suggestion, made by J. Finkelstein to one of us

(B. K.) long ago, that perhaps Regge exchanges should be compared, not at a common momentum transfer, but at a common value of trajectory α. From our point of view, a basic motivation for the hypothesis that Regge residues obey unbroken SU(3) is the observation that particle couplings and reduced resonance widths do.[3] This observation tells us that Regge residues obey SU(3) when the corresponding trajectories are passing through the physical particle poles. For example, the residues β_N and β_S of the trajectories α_N and α_S in (1.1) are SU(3)-related at the two points indicated in Fig. 2. Because $M_S^2 \neq M_N^2$, these points do not correspond to a common value of u. If, now, one hypothesizes that meson and baryon Regge residues continue to obey SU(3) as one goes away from the physical particle poles, then presumably it is not residues at a common momentum transfer, t or u, which are related, but residues at a common value of trajectory, α. Only in the latter case do the relations reduce to the observed coupling symmetry when one returns to physical J.

Consider, then, two reactions, one of which is governed by exchange of α_N and the other by exchange of α_S. In view of what we have just observed concerning residues, it is simplest, just as Finkelstein suggested, to compare the reactions at different values of u, u_N and u_S, such that $\alpha_N(u_N) = \alpha_S(u_S)$. Assuming parallel trajectories, $u_S - u_N = M_S^2 - M_N^2$. If one compares at u_N and u_S, the residues β_N and β_S will be related by Clebsch-Gordan coefficients, and all trajectory-dependent factors, such as $(s/s_o)^\alpha$, will be the same in both reactions. Thus, neglecting kinematical effects which disappear at high energy, the cross sections will be related by the Clebsch-Gordan coefficients. Apart from these, the two angular distributions are predicted to be exactly the same, except that one is shifted sideways in u relative to the other. The shift in u, $M_S^2 - M_N^2$, is a direct reflection of mass splittings, and does not disappear as we increase the energy but hold u fixed at values which are not large compared to masses. However, with the reactions compared at the properly displaced values of u, there is no exploding symmetry breaking factor $(s/s_o)^{2(\alpha_N - \alpha_S)}$.

We remark that symmetry predictions relating different reactions at different momentum transfers can have amusing consequences. If the relations hold amplitude by amplitude, then, since flip amplitudes must vanish at 0^o or 180^o, one expects that when one of these extremal points in one reaction is mapped by the symmetry into a non-extremal point in some other reaction, the flip amplitude will vanish at the latter point in the second reaction.

Predictions from exact SU(3), such as (2.1)-(2.3), relate different processes at a common energy and momentum transfer, so we have compared them in that way. But, as we see, if the processes differ as to whether strangeness is exchanged, then, as far as the contribution from a given Regge multiplet is concerned, one angular distribution is displaced in u relative to the other. The amount of displacement can vary from multiplet to multiplet, depending on mass splittings. If we compare the cross sections at one u, the residues involved are not the properly u-displaced ones which are SU(3)-related. However, if the scale constant s_o is judiciously chosen so that the factor $(s/s_o)^{\alpha(u)}$ absorbs most of the u-dependence of a Regge contribution and the residues β vary slowly with u, this is not too serious.[17] More important is the fact that at a common momentum transfer, the trajectory-dependent factors in the two reactions differ. We have, most notably, $(s/s_o)^{\alpha_N(u)}$ in one and $(s/s_o)^{\alpha_S(u)}$ in the other, leading to the symmetry breaking factor (1.1). This factor may be viewed as a simple reflection of the same characteristic Regge energy-dependence which is responsible for Regge shrinkage. What is really going on is that one angular distribution is displaced relative to the other. If one insists on comparing the two cross sections at one u, this is equivalent to comparing one of the cross sections with itself at two different values of u. At each of these u-values, this one cross section is falling as some power of the energy, but a <u>different</u> power, $2\alpha(u)-2$, at each u. Thus, the ratio of this cross section to itself at the two u-values, or, equivalently, the ratio between the cross sections for the two reactions at one u-value, will go to infinity as the energy increases.

IV. SUMMARY

We have compared a number of cross section relations predicted by unbroken SU(3) symmetry, and two predicted by exact $SU(6)_{W,\mathrm{strong}}$, with experiment. It is found that the observed cross sections conform to the symmetry predictions, apart from deviations which follow the pattern and the rough size of symmetry breaking expected from Regge pole exchange. The symmetry breaking factor $(s/s_o)^{2(\alpha_N(u) - \alpha_S(u))}$ has the interesting feature that ratios between certain SU(3)-related cross sections, compared at a common value of momentum transfer, are predicted to <u>grow</u> as the energy increases. As we have noted, this factor could fail quantitatively, but even if it should, it illustrates the fact that symmetry breaking may very well <u>not</u> disappear as the energy goes to infinity.

ACKNOWLEDGMENTS

We all wish to thank the National Accelerator Laboratory for its hospitality. Two of us (B.K. and S.M.) are also grateful for hospitality at the Aspen Center for Physics, and one of us (S.M.) for hospitality at Argonne National Laboratory.

FOOTNOTES

1. C. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Letters 1, 44 (1962).
2. S. Meshkov, C. Levinson, and H. J. Lipkin, Phys. Rev. Letters 10, 361 (1963), and S. Meshkov, G. Snow, and G. B. Yodh, Phys. Rev. Letters 13, 212 (1964).
3. B. Kayser and F. Hayot, Phys. Rev. D 6, 2423 (1972), and B. Kayser, Phys. Rev. D7, 3358 (1973), and references cited in these papers.
4. Data for $\pi^- p \to \Sigma^- K^+$ from O. Dahl et al., Phys. Rev. 163, 1430 (1967).
5. Data for $K^- n \to \Xi^- K^o$ from J. Scheuer et al. (SABRE collaboration), Nucl. Phys. B33, 61 (1971).
6. Data for $K^- p \to \Sigma^- \pi^+$ from J. Badier et al., Saclay report CEA-R3037. Higher energy data have been reported for this process by L. Moscoso et al., Nucl. Phys. B36, 332 (1972). The latter results confirm the general shape of the angular distribution, and, if Regge pole energy-dependence is assumed, imply an integrated backward cross section for 2.6 GeV/c consistent with that obtained directly from the data at that energy.
7. Data for $K^- p \to \Xi^o K^o$ from P. Dauber et al., Phys. Rev. 179, 1262 (1969).
8. P. Dauber et al., Phys. Letters 29B, 609 (1969). For the reaction $\pi^+ n \to \Delta^{++} \pi^-$, we have referred to the corresponding Lawrence Berkeley Laboratory preprint, UCRL-18789, where the data are presented separately for 1.9, 2.1, and 2.4 GeV/c.
9. The reactions of Tables I-II have also recently been considered by G. Trilling, Nucl. Phys. B40, 13 (1972). His treatment of these processes, which emphasized lower energy data, succeeded in accounting for observed symmetry breaking by a mechanism based on consideration of direct channel resonances.
10. H. Harari and H. J. Lipkin, Phys. Rev. Letters 13, 208 (1964).
11. S. Meshkov and G. B. Yodh, Phys. Rev. Letters 18, 474 (1967).

12. V. Chabaud et al., Phys. Letters 38B, 445 (1972), and D. Owen et al., Phys. Rev. 181, 1794 (1969).

13. V. Barger, F. Halzen, and M. Olsson, Nucl. Phys. B49, 206 (1972).

14. J. Carter et al., Phys. Rev. Letters 15, 373 (1965), and erratum, Phys. Rev. Letters 15, 768 (1965).

15. Data for $K^-p \to \Sigma^-\pi^+$ from L. Moscoso et al., Ref. 6.

16. Data for $K^-p \to \Xi^-K^+$ from R. Eisner et al., Phys. Rev. D7, 1 (1973).

17. Thus, in particular, analyses performed in Ref. 3 do not immediately become invalid.

TABLE I. Integrated backward cross sections for two-body reactions at 2.6 GeV/c. Data from Refs. 4-7.

Reaction Number	Reaction	Integrated Backward Cross Section (μb)	Strangeness Exchange
1	$\pi^- p \to \Sigma^- K^+$	24 ± 6	Yes
2	$K^- n \to \Xi^- K^o$	38 ± 8	Yes
3	$K^- p \to \Sigma^- \pi^+$	18 ± 5	No
4	$K^- p \to \Xi^o K^o$	6 ± 2	Yes

TABLE II. Backward cross sections for quasi two-body reactions at 2 GeV/c. In each reaction, the final baryon is a member of the $3/2^+$ decuplet. Experimental cross sections are listed in the middle column; the cross sections given in the final column have been corrected to compensate for suppression of strange exchange. Data from Ref. 8.

Reaction Number	Reaction	Backward Cross Section ($\mu b/sr$)	Strangeness Exchange	Corrected Cross Section ($\mu b/sr$)
5	$\pi^+ n \rightarrow \Delta^{++}\pi^-$	16 ± 4[a]	No	16 ± 4[a]
6	$K^- p \rightarrow Y^{*-}\pi^+$	19 ± 3	No	19 ± 3
7	$\pi^- p \rightarrow Y^{*-}K^+$	4.5 ± 1	Yes	13.5 ± 3
8	$K^- p \rightarrow \Xi^{*-}K^+$	3^{+2}_{-1}	Yes	9^{+6}_{-3}

a. For reaction 5, we quote 1/3 of the measured cross section.

<u>TABLE III.</u> Cross sections at 180° and 4 GeV/c for reactions related by $SU(6)_{W,\text{strong}}$. Data from Refs. 4, 15, and 16.

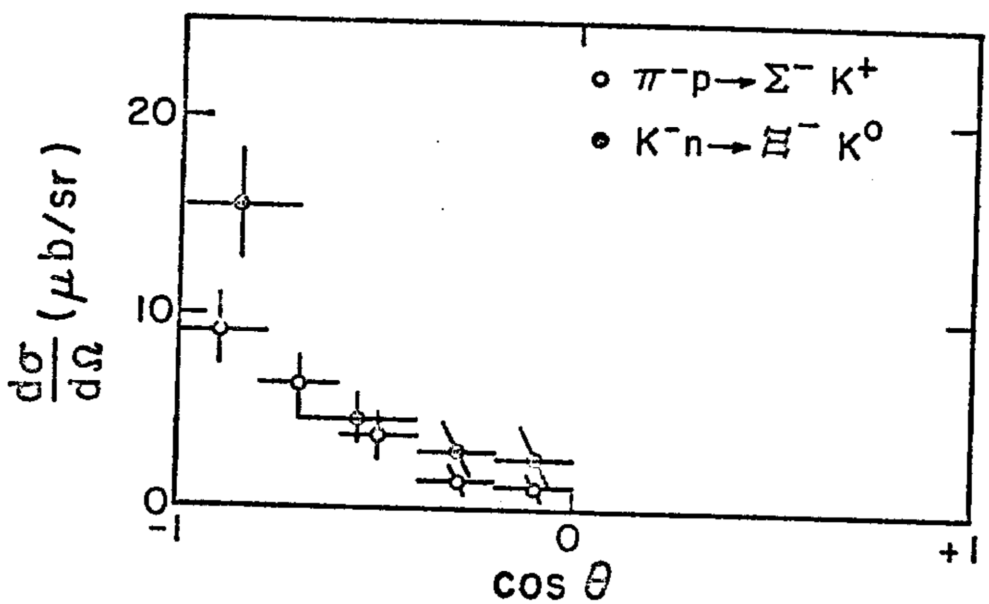
Reaction Number	Reaction	Cross Section at 180° ($\mu b/GeV^2$)	Strangeness Exchange
1	$\pi^- p \to \Sigma^- K^+$	8 ± 3	Yes
9	$K^- p \to \Xi^- K^+$	6 ± 2[a]	Yes
3	$K^- p \to \Sigma^- \pi^+$	18 ± 5	No

a. For reaction 9, we quote 1/4 of the measured cross section.

FIGURE CAPTIONS

1. Backward angular distributions for $\pi^- p \to \Sigma^- K^+$ and $K^- n \to \Xi^- K^0$ at 2.6 GeV/c. Data from Refs. 4 and 5.

2. The non-strange and strange trajectories, α_N and α_S, plotted against u. The residues β_N and β_S correspond to ordinary particle couplings at the two darkly circled points, where the trajectories pass through the spin J of the physical particles.



$\pi^- p \rightarrow \Sigma^- K^+$
$K^- n \rightarrow \Xi^- K^0$
20
10
0
$\frac{d\sigma}{d\Omega}$ (μb/sr)
-1
0
+1
cos θ

FIG. 1

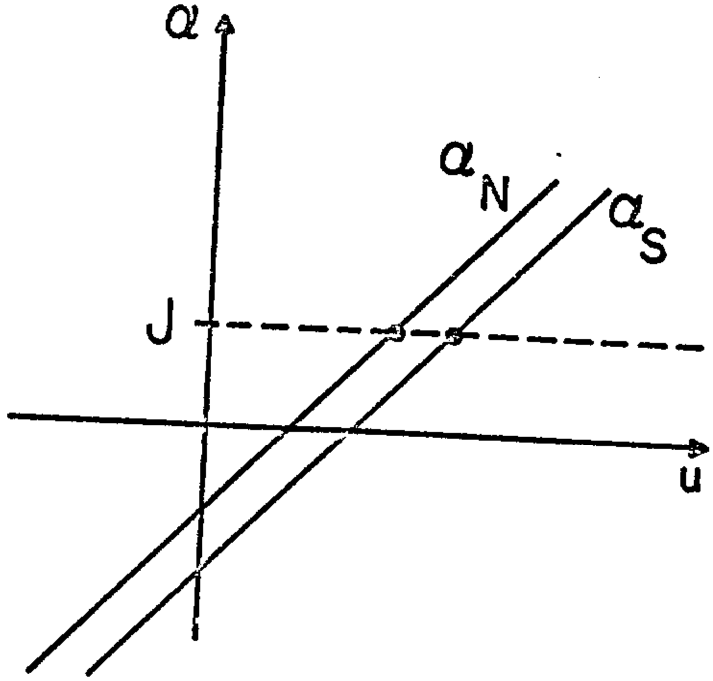


FIG. 2